Key Issues with the K_{is} Term in Generator Models GENTPJ and GENQEJ

This note highlights two primary concerns introduced by the inclusion of the K_{is} term in synchronous generator models such as GENTPJ and GENQEJ:

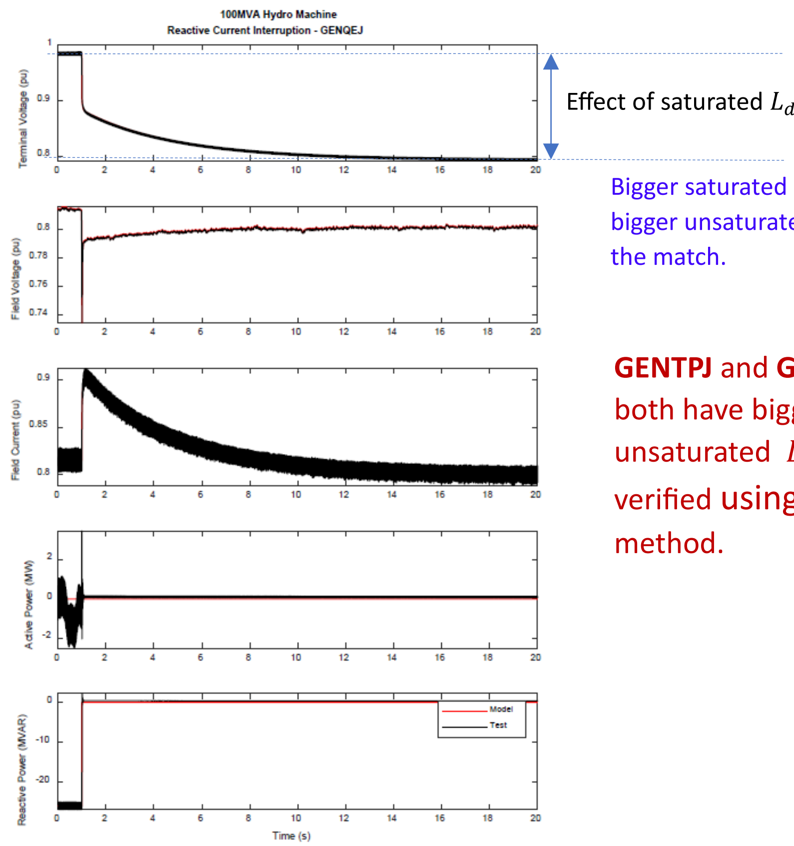
1. **Inflated Unsaturated D-axis Reactance (X_d) Verification Result**
   When the stator current decrement method (D-axis load rejection test) is used to verify the unsaturated d-axis reactance, the presence of a positive non-zero K_{is} artificially increases the value of this parameter. (X_d and L_d are used interchangeably hereafter.)

2. **Non-continuous Field Current Rate of Change with Respect to Stator D-axis Current**
   With zero active power, in the vicinity of zero reactive power (i_d crossing from negative to positive), the K_{is}-dependent saturation term produces a sudden change in the slope of the field-current-versus-d-axis-current characteristic. This means the model becomes non-continuous at Q = 0, unlike all other saturation models used in power system stability studies where a smooth (at least first- and second-order continuous) behavior is expected. This non-continuity may impact numerical stability, sensitivity-based methods, and the consistency of parameter identification.

These two characteristics introduce modeling behavior that may not be physically representative and may complicate both parameter verification and stability study. Using the example unit of the **GENQEJ** presentation (the Presentation) in the MVS meeting on September 11, 2025, the following analysis provides supporting equations, examples, and test evidence to illustrate these effects.

1. $K_{is} > 0$ yields bigger L_d in stator current decrement test

genqej
4.8300 tpdo
0.0320 tppdo
0.2000 tpqo
0.0420 tppqo
3.9550 h
0.0000 d
0.8100 ld
0.6600 lq
0.3950 lpd
0.5100 lpq
0.3100 lppd
0.3100 lppq
0.2000 ll
0.0670 s1
0.3000 s12
0.0016 ra
0.0000 rcomp
0.0000 xcomp
0.5000 accel
0.0850 kis
0.0000 sk



Bigger saturated factor, bigger unsaturated L_d for the match.

GENTPJ and **GENQEJ** both have bigger unsaturated L_d when verified using this method.

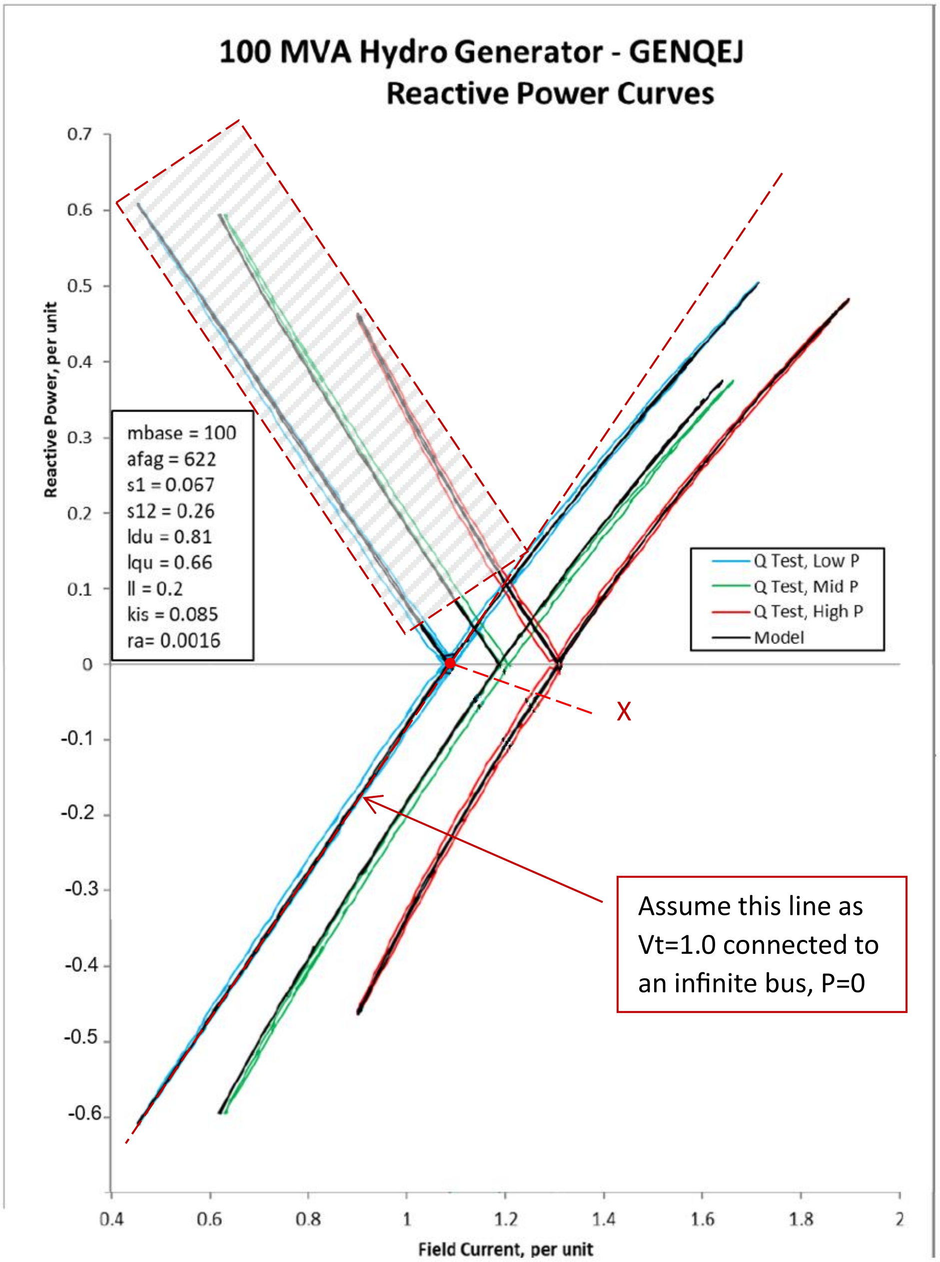
Plot above is taken from Page 13 of the Presentation for GENQEJ. The voltage drop in the D-axis load rejection test reflects the combined effect of an unsaturated L_d and the saturation effect. The application of K_{is} is to universally increase the degree of saturation even in under-excited operating conditions. Consequently, the unsaturated L_d verified using GENTPJ and GENQEJ through this d-axis load rejection is bigger than other second-order synchronous machine models.

Detail calculations using this example unit are given in Annex A.

2. $K_{is} > 0$ Causes Non-continuous Field Current Rate of Change when Stator Current Changes from Capacitive to Inductive

The low power curve on Page 12 of the Presentation ideally represents a generator connected to infinite bus with 0 MW output, with reactive power changed from negative to positive.

When reactive power is plotted as both positive and negative, like in the plot below, mathematically **GENTPJ** and **GENQEJ** will have a sudden slope change at point X (Q=0) caused by a non-zero K_{is} in the saturation function. Detail description is given in Annex B.



Annex A:

From **GENTPJ** equations and the proposed **GENQEJ** model block diagram, in the steady-state, we have below equation.

$$(L_{ad}i_{fd}) = V_q(1 + f_{sat}) + L_d i_d + f_{sat} L_l i_d$$

From the plot on Page 2, below measurement values can be estimated for the initial condition:

$P = 0;\quad V_t = V_q = 0.98;\quad Q = -26\ Mvar$. Calculated $i_d = -0.265\ pu$

Use exponential saturation as an example. With given $S_{1.0} = 0.067$ and $S_{1.2} = 0.30$, calculated $A = 8.222$, $B = 0.067$.

$$f_{sat} = 0.067(x)^{8.222}$$

where $x = V_q + L_l i_d + K_{is}|i_d|$ for the operating conditions being analyzed here.

The saturation factor for the initial condition (before d-axis current is interrupted) is:

$$f_{sat} = 0.067(0.98 - 0.2 * 0.265 + 0.085 * 0.265)^{8.222} = 0.0438$$

Field current

$$(L_{ad}i_{fd}) = V_q(1 + f_{sat}) + L_d i_d + f_{sat} L_l i_d$$
$$= 0.98 * 1.0438 - 0.81 * 0.265 - 0.0438 * 0.2 * 0.265 = 0.806$$

This calculated initial field current per unit value is in line with the field current plot on Page 2.

When this test case is used to verify L_d using other second-order models, the difference of the saturation function will lead different results. Below table list 4 cases being considered.

#	Field Current Equation	f_{sat} input x	Model or method
1	$(L_{ad}i_{fd}) = V_q(1 + f_{sat}) + L_d i_d + f_{sat} L_l i_d$	$V_q + L_l i_d$	GENQEC w/o Kw GENTPJ w/o Kis
2	$(L_{ad}i_{fd}) = V_q(1 + f_{sat}) + L_d i_d + f_{sat} L''_d i_d$	$V_q + L''_d i_d$	GENROU/GENROE
3	$(L_{ad}i_{fd}) = V_q(1 + f_{sat}) + L_d i_d + f_{sat} L'_d i_d$	$V_q + L'_d i_d$	GENSAL/GENSAE
4	$(L_{ad}i_{fd}) = V_q(1 + f_{sat}) + L_d i_d + f_{sat} X_p i_d$	$V_q + X_p i_d$	Potier Reactance

With the parameters given in the plot on Page 2, for case #1,

$$f_{sat} = 0.067(0.98 - 0.2 * 0.265)^{8.222} = 0.0359$$

$$0.806 = V_q(1 + f_{sat}) + L_d i_d + f_{sat} L_l i_d = 0.98 * 1.0359 - L_d * 0.265 - 0.0359 * 0.2 * 0.265$$

$$L_d = \frac{(0.98 * 1.0359 - 0.0359 * 0.2 * 0.265 - 0.806)}{0.265} = 0.782$$

Case #2,

$$f_{sat} = 0.067(0.98 - 0.31 * 0.265)^{8.222} = 0.0276$$

$$L_d = \frac{(0.98 * 1.0276 - 0.0276 * 0.31 * 0.265 - 0.806)}{0.265} = 0.750$$

Case #3,

$$f_{sat} = 0.067(0.98 - 0.3950 * 0.265)^{8.222} = 0.0224$$

$$L_d = \frac{(0.98 * 1.0224 - 0.0224 * 0.395 * 0.265 - 0.806)}{0.265} = 0.731$$

Case #4,

Potier Reactance $X_p > X_l$ where X_l is numerically the same as L_l at synchronous speed in steady-state. Compared with the calculation in Case #1, when Potier reactance is used, we will have $f_{sat} < 0.0359$ and $L_d < 0.782$.

Noting the application of X_p in over-excited area is overlapped with K_{is} to increase the degree of saturation, assuming X_p would have the same or similar performance as K_{is} on this zero power factor line, $X_p = X_l + K_{is}$.

$$f_{sat} = 0.067(0.98 - 0.285 * 0.265)^{8.222} = 0.0293$$

$$L_d = \frac{(0.98 * 1.0293 - 0.0293 * 0.285 * 0.265 - 0.806)}{0.265} = 0.757$$

Conclusion:

When using stator decrement method to verify generator model parameters, the use of $K_{is}I_t$ term will result in a systematically higher L_d (i.e., unsaturated X_d) compared with other second-order synchronous machine models. The $K_{is}I_t$ term works to the contrary of the common understanding that in such under-excited working condition, stator current is assisting the rotor MMF to create the main flux across the air-gap.

Annex B:

The steady-state field current in both **GENTPJ** and **GENQEJ** model can be calculated as

$$\left(L_{ad} i_{fd}\right) = V_q(1 + f_{sat}) + L_d i_d + f_{sat} L_l i_d$$

With the ideal MW=0 and V_t=1.0 pu case, the "low P" Q-test plot is mathematically the field current $\left(L_{ad} i_{fd}\right)$ versus stator d-axis current i_d. For the general case with infinite bus connected, V_q is independent of i_d. The slope of the "low P" plot can be calculated as:

$$\frac{\partial\left(L_{ad} i_{fd}\right)}{\partial i_d} = L_d + V_q \frac{\partial(f_{sat})}{\partial i_d} + f_{sat} L_l + L_l i_d \frac{\partial(f_{sat})}{\partial i_d}$$

With $P = 0$, and saturation function example is calculated as in Annex A,

$$f_{sat} = 0.067(V_q + \ L_l i_d + K_{is}|i_d|)^{8.222}$$

V_t is a constant, independent of i_d, $V_q = V_t$, the slope of the saturation function at $i_d = 0$ is:

$$\left.\frac{\partial(f_{sat})}{\partial i_d}\right|_{i_d=0} = \left.\left(8.222 * 0.067(V_q + \ L_l i_d + K_{is}|i_d|)^{7.222}(L_l \pm K_{is})\right)\right|_{i_d=0}$$

The sign before K_{is} depends on whether i_d is approaching zero from positive or negative $(0^+ \ or \ 0^-)$, because the absolute value of i_d is multiplied to K_{is}. Assuming $V_t = V_q = 1.0$,

$$\left.\frac{\partial(f_{sat})}{\partial i_d}\right|_{i_d=0^+} = \left.\left(8.222 * 0.067(1 + \ L_l i_d + K_{is}|i_d|)^{7.222}(L_l + K_{is})\right)\right|_{i_d=0^+} = 0.157$$

$$\left.\frac{\partial(f_{sat})}{\partial i_d}\right|_{i_d=0^-} = \left.\left(8.222 * 0.067(1 + \ L_l i_d + K_{is}|i_d|)^{7.222}(L_l - K_{is})\right)\right|_{i_d=0^-} = 0.0634$$

With $V_q = 1.0$, the reciprocal of the slope at point X in the plot on page 3 can be calculated as

$$\left.\frac{\partial\left(L_{ad} i_{fd}\right)}{\partial i_d}\right|_{i_d=0} = \left.\left(L_d + V_q \frac{\partial(f_{sat})}{\partial i_d} + f_{sat} L_l + L_l i_d \frac{\partial(f_{sat})}{\partial i_d}\right)\right|_{i_d=0} = L_d + \left.\frac{\partial(f_{sat})}{\partial i_d}\right|_{i_d=0} + f_{sat} L_l$$

$$= 0.81 + \begin{pmatrix} 0.157 \\ 0.0634 \end{pmatrix} + 0.067 * 0.2 = \begin{pmatrix} 0.980 \\ 0.887 \end{pmatrix}$$

<u>Conclusion:</u>

The inclusion of the $K_{is} I_t$ term in the saturation function causes the first order derivative of the saturation non-continuous with a $K_{is} > 0$. This is distinctively different from all other methods of including saturation effect in second-order synchronous machine dynamic models. This behavior can theoretically affect numerical sensitivity, parameter identification, and the consistency of dynamic simulation. The impact of this has not been fully studied.