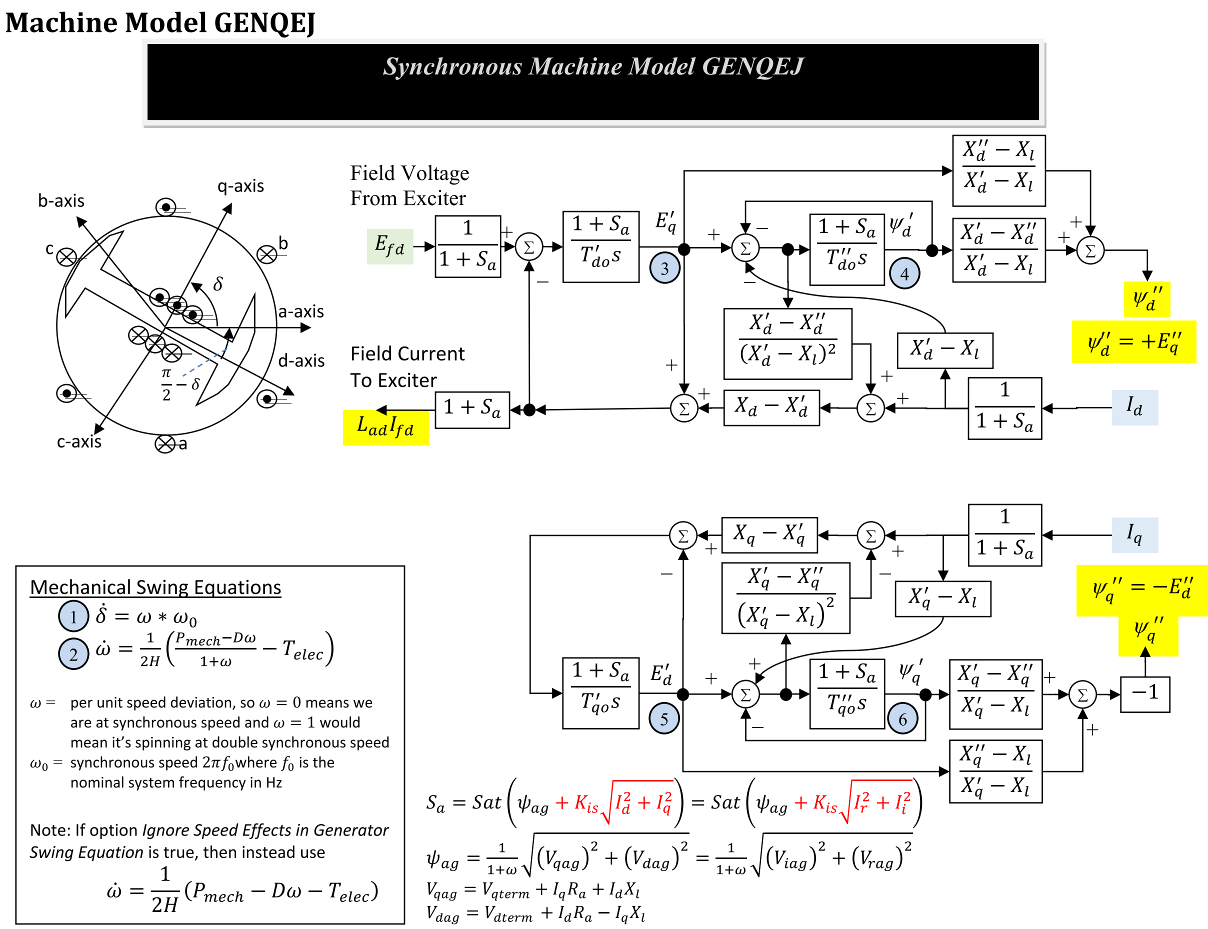
Machine Model: GENQEJ

AutoCorrection Properties

Model Equations and/or Block Diagrams



Parameters:

H	Inertia constant, sec
D	Damping factor, pu
Ra	Stator resistance, pu
Xd	Direct axis synchronous reactance
Xq	Quadrature axis synchronous reactance
Xdp	Direct axis transient reactance
Xqp	Quadrature axis transient reactance
Xdpp	Direct axis subtransient reactance
Xqpp	Quadrature axis subtransient reactance
Xl	Stator leakage reactance
Tdop	Open circuit direct axis transient time constant
Tqop	Quadrature axis transient time constant
Tdopp	Open circuit direct axis subtransient time constant
Tqopp	Quadrature axis subtransient time constant
S1	Saturation factor at 1.0 pu flux
S12	Saturation factor at 1.2 pu flux
RComp	Compensating resistance for voltage control, pu
XComp	Compensating reactance for voltage control, pu
Accel	Acceleration factor
Kis	Current multiplier for saturation calculation
SatFunc	SatFunc; 0 = exponential; 1 = Scaled Quadratic; 2 = Quadratic

The GENQEC model includes saturation not simply as an additive term to the derivative of the E'_d and E'_q terms, but instead as the saturation of all input parameters directly (impedance, time constants, field voltage, and field current). This means that the network interface equations and Electrical Torque equations are impacted by the saturation as well. The relationships are as described in the following.

Saturated Impedance Terms

$S_a = at\left(\psi_{ag} + K_{is}\sqrt{I_d^2 + I_q^2}\right) = Sat\left(\psi_{ag} + K_{is}\sqrt{I_r^2 + I_i^2}\right)$

$\psi_{ag} = \frac{1}{1+\omega}\sqrt{\left(V_{qag}\right)^2 + \left(V_{dag}\right)^2}$

$V_{qag} = V_{qterm} + I_q R_a + I_d X_l$ (air gap voltage)

$V_{dag} = V_{dterm} + I_d R_a - I_q X_l$ (air gap voltage)

$X''_{dsat} = \dfrac{X''_d - X_l}{1 + S_a} + X_l$

$X''_{qsat} = \dfrac{X''_q - X_l}{1 + S_a} + X_l$

Electrical Torque

$\psi_q = \psi_q'' - I_q X''_{qsat}$

$\psi_d = \psi_d'' - I_d X''_{dsat}$

$T_{elec} = \psi_d I_q - \psi_q I_d$

Network Interface Equations

$\boldsymbol{V} = \frac{d\boldsymbol{\psi}}{dt} = j(1+\omega)\left(\psi_d'' + j\psi_q''\right)$

$V_d + jV_q = \left(-\psi''_q + j\psi''_d\right)(1+\omega)$

$V_d + jV_q = \left(E''_d + jE''_q\right)(1+\omega)$

Flag Parameter determines what saturation function is used

0 : Exponential $Sat(x) = Bx^A$

1 : Scaled Quadratic $Sat(x) = \begin{cases} B(x-A)^2/x & If\ x > A \\ 0 & If\ x \le A \end{cases}$

2 : Quadratic $Sat(x) = \begin{cases} B(x-A)^2 & If\ x > A \\ 0 & If\ x \le A \end{cases}$

Else we assume Exponential

Solid Rotor Machine

Time Constants must be greater than zero

Reactances must be greater than zero

Salient Pole Machine with single amortisseur windings on d and q axis

(see block diagram in subsequent pages)

$T'_{qo} = 0$; $X'_q = X_q$; $T'_{do} > 0$; $T''_{qo} > 0$; $T''_{do} > 0$

Salient Pole Machine without amortisseur windings

(see block diagram in subsequent pages)

$T'_{do} > 0$; $T'_{qo} = T''_{qo} = T''_{do} = 0$; $X''_d = X'_d$; $X''_q = X'_q = X_q$

Because of sub-transient saliency ($X''_d <> X''_q$), X''_{dsat} may not be equal to X''_{qsat}. Because of this the network interface equations cannot be written as a complex number circuit equation. Instead the network equations are written as follows

$V_{dterm} = V_d - R_a I_d + X''_{qsat} I_q$

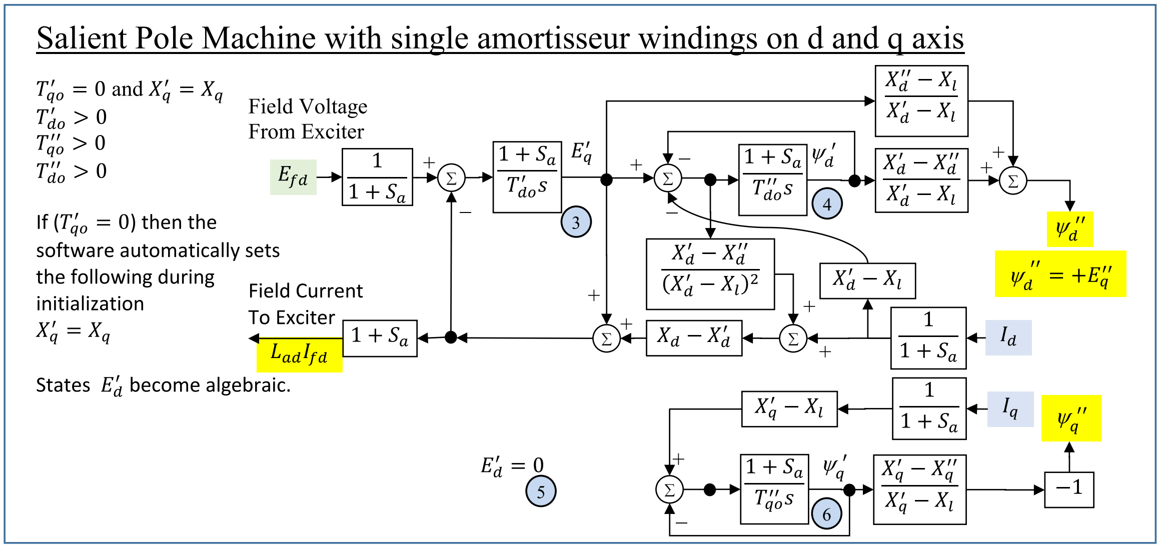
$V_{qterm} = V_q - X''_{dsat} I_d - R_a I_q$

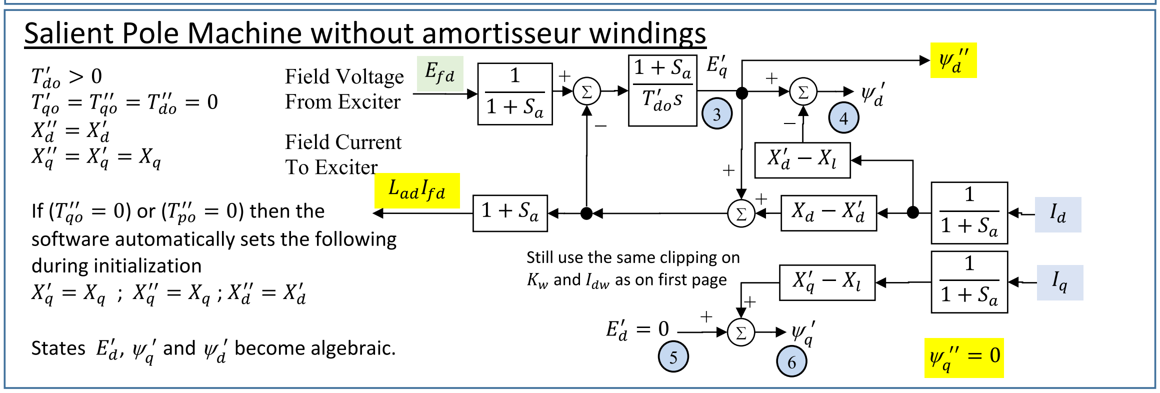
These equations must be used directly when modeling the network boundary equations

Treatment of *Rcomp* and *Xcomp*

When specified, the compensated voltage fed as an input to the exciter is calculated as:

$V_{comp} = \left|\overline{V_t} - (R_{comp} + jX_{comp})\overline{I_t}\right|$


Salient Pole Machine with single amortisseur windings on d and q axis
$T'_{qo} = 0$ and $X'_q = X_q$
$T'_{do} > 0$
$T''_{qo} > 0$
$T''_{do} > 0$
If ($T'_{qo} = 0$) then the software automatically sets the following during initialization
$X'_q = X_q$
States E'_d become algebraic.
Field Voltage From Exciter
E_{fd}
$\frac{1}{1+S_a}$
$\frac{1+S_a}{T'_{do}s}$
E'_q
$\frac{1+S_a}{T''_{do}s}$
ψ_d'
$\frac{X''_d - X_l}{X'_d - X_l}$
$\frac{X'_d - X''_d}{X'_d - X_l}$
$\frac{X'_d - X''_d}{(X'_d - X_l)^2}$
$X'_d - X_l$
ψ_d''
$\psi_d'' = +E''_q$
Field Current To Exciter
$L_{ad}I_{fd}$
$1+S_a$
$X_d - X'_d$
I_d
I_q
$X'_q - X_l$
ψ_q''
$E'_d = 0$
$\frac{1+S_a}{T''_{qo}s}$
ψ_q'
$\frac{X'_q - X''_q}{X'_q - X_l}$
-1


Salient Pole Machine without amortisseur windings
$T'_{do} > 0$
$T'_{qo} = T''_{qo} = T''_{do} = 0$
$X''_d = X'_d$
$X''_q = X'_q = X_q$
If ($T''_{qo} = 0$) or ($T''_{po} = 0$) then the software automatically sets the following during initialization
$X'_q = X_q$; $X''_q = X_q$; $X''_d = X'_d$
States E'_d, ψ_q' and ψ_d' become algebraic.
Field Voltage From Exciter
E_{fd}
$\frac{1}{1+S_a}$
$\frac{1+S_a}{T'_{do}s}$
E'_q
ψ_d''
ψ_d'
$X'_d - X_l$
Field Current To Exciter
$L_{ad}I_{fd}$
$1+S_a$
$X_d - X'_d$
I_d
Still use the same clipping on K_w and I_{dw} as on first page
$X'_q - X_l$
I_q
$E'_d = 0$
ψ_q'
$\psi_q'' = 0$

Initialization

This model is similar to GENROU, but the saturation is handled much differently. We will more formerly discuss the theory of this model in a later section. Conceptually though, the saturation function is applied to the reactances, field voltage input (E_{fd}), field current output ($L_{ad}I_{fd}$), and also the various time constants ($T'_{qo}, T''_{qo}, T'_{do}, T''_{qo}$). The model also permits transient saliency so that $X''_d <> X''_q$. The saturation function is a function of the air gap flux (ψ_{ag}). Satuation is then applied directly to both d and q axis without any scaling because we are dividing all reactance terms by the same $(1 + S_a)$ value. The saturation function is calculated from the air gap voltage magnitude on either the d/q reference frame or the network reference frame. Similar to GENROU with saturation we need to be a little clever with the initialization. We know that the initial speed deviation is $\omega = 0$, however we cannot directly solve for rotor angle. Let's just write out the 5 equations that must be satisfied for steady state.

1. $+E'_q - \psi'_d - (X'_d - X_l)\frac{I_d}{1+S_a} = 0$ Sum inputs integrator for State 4
2. $\psi''_d - \left(\frac{X''_d - X_l}{X'_d - X_l}\right)E'_q - \left(\frac{X'_d - X''_d}{X'_d - X_l}\right)\psi'_d = 0$ Final summation block resulting in ψ''_d
3. $-E'_d + \left(X_q - X'_q\right)\frac{I_q}{1+S_a} = 0$ Sum inputs integrator for State 5
4. $+E'_d - \psi'_q + \left(X'_q - X_l\right)\frac{I_q}{1+S_a} = 0$ Sum inputs integrator for State 6
5. $-\psi''_q - \left(\frac{X''_q - X_l}{X'_q - X_l}\right)E'_d - \left(\frac{X'_q - X''_q}{X'_q - X_l}\right)\psi'_q = 0$ Final summation block resulting in ψ''_q

The value of the saturation function $Sat(\psi_{ag})$ is vital to this initialization. We can get the air-gap voltage on the network reference frame and the magnitude on the d/q reference frame will be the same

- $V_{iag} = V_{iterm} + I_i R_a + I_r X_l$ (air gap voltage)
- $V_{rag} = V_{rterm} + I_r R_a - I_i X_l$ (air gap voltage)
- $\psi_{ag} = \frac{1}{1+\omega}\sqrt{\left(V_{qag}\right)^2 + \left(V_{dag}\right)^2} = \frac{1}{1+\omega}\sqrt{\left(V_{rag}\right)^2 + \left(V_{iag}\right)^2}$
- $S_a = Sat\left(\psi_{ag} + K_{is}\sqrt{I_r^2 + I_i^2}\right)$

The initial rotor angle can be solve using the following equation which is derived in a section shortly.

- $\delta = \tan^{-1}\left\{\frac{V_{iterm} + R_a I_i + I_r\left(\frac{X_q - X_l}{1+S_a} + X_l\right)}{V_{rterm} + R_a I_r - I_i\left(\frac{X_q - X_l}{1+S_a} + X_l\right)}\right\}$
- $V_d = V_r \sin(\delta) - V_i \cos(\delta)$ convert to dq axis using δ
- $V_q = V_r \cos(\delta) + V_i \sin(\delta)$ convert to dq axis using δ
- $I_d = I_r \sin(\delta) - I_i \cos(\delta)$ convert to dq axis using δ
- $I_q = I_r \cos(\delta) + I_i \sin(\delta)$ convert to dq axis using δ
- $\psi''_q = -V_d / (1 + \omega)$ really we can ignore ω as it's zero
- $\psi''_d = +V_q / (1 + \omega)$ really we can ignore ω as it's zero
- $\psi'_d = \psi''_d - (X''_d - X_l)\left(\frac{I_d}{1+S_a}\right)$ Solve Equation 1 for E'_q and substitute into Equation 2
- $E'_q = \psi'_d + (X'_d - X_l)\left(\frac{I_d}{1+S_a}\right)$ Equation 1 solve for E'_q
- $E'_d = \left(X_q - X'_q\right)\left(\frac{I_q}{1+S_a}\right)$ Equation 3 solve for E'_d
- $\psi'_q = E'_d + \left(X'_q - X_l\right)\left(\frac{I_q}{1+S_a}\right)$ Equation 4 solve for ψ'_q

Once we have solved this set of equations, then we solve for initial values of E_{fd} and $L_{ad}I_{fd}$ which are equal at the initial condition because their difference is fed into the T'_{do} integration block. This is similar to GENROU without saturation except the yellow terms below. Notice the extra $(1-K_w I_{dw})$ term.

$$E_{fd} = L_{ad}I_{fd} = \left(\frac{1+S_a}{1}\right)\left\{+E'_q + (X_d - X'_d)\left[\frac{I_d}{1+S_a} + \left(\frac{X'_d - X''_d}{(X'_d - X_l)^2}\right)\left(+E'_q - \psi'_d - (X'_d - X_l)\frac{I_d}{1+S_a}\right)\right]\right\}$$

This document will not show the algebra, but you can also greatly simply this equation at steady state to $E_{fd} = L_{ad}I_{fd} = \left(\frac{1+S_a}{1}\right)\left\{V_{qterm} + \left(\frac{X_d - X_l}{1+S_a} + X_l\right)I_d + R_a I_q\right\}$ which is only a function of X_d and X_l.

1.1.1 Initial Rotor Angle Derivation

It is possible to get a closed form solution of these equations as follows.

Use equation 4 to solve for	$\psi'_q = (X'_q - X_l)\left(\frac{I_q}{1+S_a}\right) + E'_d$
Substitute this into equation 5	$-\psi''_q - \left(\frac{X''_q - X_l}{X'_q - X_l}\right)E'_d - \left(\frac{X'_q - X''_q}{X'_q - X_l}\right)\left[(X'_q - X_l)\left(\frac{I_q}{1+S_a}\right) + E'_d\right] = 0$
Expand this to	$-\psi''_q - \left(\frac{X''_q - X_l}{X'_q - X_l}\right)E'_d - \left(\frac{X'_q - X''_q}{X'_q - X_l}\right)E'_d - \left(\frac{X'_q - X''_q}{X'_q - X_l}\right)(X'_q - X_l)\left(\frac{I_q}{1+S_a}\right) = 0$
Group E'_d terms to simply to	$-\psi''_q - E'_d - (X'_q - X''_q)\left(\frac{I_q}{1+S_a}\right) = 0$
Flip signs to get	$+\psi''_q + E'_d + (X'_q - X''_q)\left(\frac{I_q}{1+S_a}\right) = 0$
Use equation 3 to solve for	$E'_d = +(X_q - X'_q)\left(\frac{I_q}{1+S_a}\right)$
Substite this into previous equation to get	$+\psi''_q + (X_q - X'_q)\left(\frac{I_q}{1+S_a}\right) + (X'_q - X''_q)\left(\frac{I_q}{1+S_a}\right) = 0$
This then simplifies to	$+\psi''_q + (X_q - X''_q)\left(\frac{I_q}{1+S_a}\right) = 0$
Network equation for V_d	$\psi''_q = -V_d = -V_{dterm} - R_a I_d + X''_{qsat} I_q$
This becomes	$-V_{dterm} - R_a I_d + X''_{qsat} I_q + (X_q - X''_q)\frac{I_q}{(1+S_a)} = 0$
Substitute in the value of X''_{qsat}	$-V_{dterm} - R_a I_d + \left(\frac{X''_q - X_l}{1+S_a} + X_l\right)I_q + \frac{(X_q - X''_q)}{(1+S_a)}I_q = 0$
This then simplifies to	$-V_{dterm} - R_a I_d + \left(\frac{X_q - X_l}{1+S_a} + X_l\right)I_q = 0$
Convert them to the network reference and define X_{qsat}	$I_q = I_r\cos(\delta) + I_i\sin(\delta)$; $I_d = I_r\sin(\delta) - I_i\cos(\delta)$ $V_{dterm} = V_{rterm}\sin(\delta) - V_{iterm}\cos(\delta)$; $X_{qsat} = \left(\frac{X_q - X_l}{1+S_a} + X_l\right)$
This gives us	$-[V_{rterm}\sin(\delta) - V_{iterm}\cos(\delta)] - R_a[I_r\sin(\delta) - I_i\cos(\delta)]$ $+X_{qsat}[I_r\cos(\delta) + I_i\sin(\delta)] = 0$
Group the sine and cosine terms and move sine terms	$[V_{rterm} + R_a I_r - I_i X_{qsat}]\sin(\delta) = [V_{iterm} + R_a I_i + I_r X_{qsat}]\cos(\delta)$
Turn this into the tangent function	$\frac{\sin(\delta)}{\cos(\delta)} = \tan(\delta) = \frac{V_{iterm} + R_a I_i + I_r X_{qsat}}{V_{rterm} + R_a I_r - I_i X_{qsat}}$

This gives us our steady state rotor angle based only on the network reference frame terminal voltage and current and the saturation value which can be calculated from terminal voltage.

$$\delta = \tan^{-1}\left\{\frac{V_{iterm} + R_a I_i + I_r\left(\frac{X_q - X_l}{1+S_a} + X_l\right)}{V_{rterm} + R_a I_r - I_i\left(\frac{X_q - X_l}{1+S_a} + X_l\right)}\right\}$$

Writing this way is nice, because you can see that when the parameter $S_a = 0$ (meaning no saturation) this simplifies to exactly what we had in GENROU without saturation.

$$\delta = \tan^{-1}\left(\frac{V_{iterm} + R_a I_i + X_q I_r}{V_{rterm} + R_a I_r - X_q I_i}\right)$$

Theory of model: Modification of Saturation Model used in GENROU

The improvements with the GENQEC are similar as for GENTPF and GENTPJ models and are primarily related to the treatment of saturation. It has however been noticed that the dynamic equations (block diagram) in GENTPF and GENTPJ are simplified from the GENROU resulting a small change in transient behavior. To avoid this, let's step back to the complete GENROU model and add a similar treatment as done for GENTPF for saturation.

Application of Saturation to the block diagram terms

To see where this comes from, let's go back to the book *Power System Dynamics and Stability* by Peter Sauer and M.A. Pai and look at equations the derivation of the equations that led to the GENROU/GENSAL models. In particular, page 39 and 40 of that book and equations 3.120 and 3.130 – 3.139. A scanned image of these equations is shown in the follow image.

Orange terms do not saturate

Blue terms do saturate

$$X_d \triangleq X_{\ell s} + X_{md}, \quad X_q \triangleq X_{\ell s} + X_{mq} \tag{3.120}$$

$$X'_d \triangleq X_{\ell s} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{\ell fd}}} = X_{\ell s} + \frac{X_{md}X_{\ell fd}}{X_{fd}} = X_d - \frac{X_{md}^2}{X_{fd}} \tag{3.130}$$

$$X'_q \triangleq X_{\ell s} + \frac{1}{\frac{1}{X_{mq}} + \frac{1}{X_{\ell 1q}}} = X_{\ell s} + \frac{X_{mq}X_{\ell 1q}}{X_{1q}} = X_q - \frac{X_{mq}^2}{X_{1q}} \tag{3.131}$$

$$X''_d \triangleq X_{\ell s} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{\ell fd}} + \frac{1}{X_{\ell 1d}}} \tag{3.132}$$

$$X''_q \triangleq X_{\ell s} + \frac{1}{\frac{1}{X_{mq}} + \frac{1}{X_{\ell 1q}} + \frac{1}{X_{\ell 2q}}} \tag{3.133}$$

$$T'_{do} \triangleq \frac{X_{fd}}{\omega_s R_{fd}} \tag{3.134}$$

$$T'_{qo} \triangleq \frac{X_{1q}}{\omega_s R_{1q}} \tag{3.135}$$

$$T''_{do} \triangleq \frac{1}{\omega_s R_{1d}} \left(X_{\ell 1d} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{\ell fd}}} \right) \tag{3.136}$$

$$T''_{qo} \triangleq \frac{1}{\omega_s R_{2q}} \left(X_{\ell 2q} + \frac{1}{\frac{1}{X_{mq}} + \frac{1}{X_{\ell 1q}}} \right) \tag{3.137}$$

$$E_{fd} \triangleq \frac{X_{md}}{R_{fd}} V_{fd} \tag{3.139}$$

Taking a closer look at all the terms boxed in blue above, going back a page in the Sauer/Pai book are the following definitions.

$$X_{\ell s} \triangleq \frac{\omega_s L_{\ell s}}{Z_{BDQ}}, \quad X_{md} \triangleq \frac{\omega_s L_{md}}{Z_{BDQ}}, \quad X_{mq} \triangleq \frac{\omega_s L_{mq}}{Z_{BDQ}} \qquad (3.115)$$

$$X_{\ell fd} \triangleq X_{fd} - X_{md}, \quad X_{\ell 1d} \triangleq X_{1d} - X_{md} \qquad (3.118)$$

$$X_{\ell 1q} \triangleq X_{1q} - X_{mq}, \quad X_{\ell 2q} \triangleq X_{2q} - X_{mq} \qquad (3.119)$$

These terms then refer to the green and ultimately red boxed terms in the following equations.

$$X_{fd} \triangleq \frac{\omega_s L_{fdfd}}{Z_{BFD}}, \quad X_{1d} \triangleq \frac{\omega_s L_{1d1d}}{Z_{B1D}}, \quad X_{fd1d} \triangleq \frac{\omega_s L_{fd1d} L_{sfd}}{Z_{BFD} L_{s1d}} \qquad (3.116)$$

$$X_{1q} \triangleq \frac{\omega_s L_{1q1q}}{Z_{B1Q}}, \quad X_{2q} \triangleq \frac{\omega_s L_{2q2q}}{Z_{B2Q}}, \quad X_{1q2q} \triangleq \frac{\omega_s L_{1q2q} L_{s1q}}{Z_{B1Q} L_{s2q}} \qquad (3.117)$$

$$L_{md} \triangleq \frac{3}{2}(L_A + L_B), \quad L_{mq} \triangleq \frac{3}{2}(L_A - L_B) \qquad (3.105)$$

$$L_{rr}(\theta_{\text{shaft}}) \triangleq \begin{bmatrix} L_{fdfd} & L_{fd1d} & 0 & 0 \\ L_{fd1d} & L_{1d1d} & 0 & 0 \\ 0 & 0 & L_{1q1q} & L_{1q2q} \\ 0 & 0 & L_{1q2q} & L_{2q2q} \end{bmatrix} \qquad (3.97)$$

Ultimately, all our reactances, time constants, and even the scaled field voltage and current end up being directly proportional to the following red-boxed terms.

- L_{md} : d-axis stator inductance
- L_{mq} : q-axis stator inductance
- L_{fdfd}, L_{1d1d} : d-axis rotor inductances
- L_{1q1q}, L_{2q2q} : q-axis rotor inductances

All of these six inductance terms represent inductance inside the windings of the synchronous machine. Thus all of these terms are going to experience saturation because their flux path is through the iron core. There is also dependence on the leakage inductance L_{ls} (boxed in purple above). The leakage inductance does not saturate because it is dominated by the characteristic of the air gap and we assume that the flux in the air does not saturate.

To derive the GENQEC model, assume all d-axis inductances are scaled by the Sat_d term and all q-axis inductances are scaled by the Sat_q term as follows.

$$L_{mdsat} = \frac{L_{md}}{Sat_d} \qquad L_{fdfdsat} = \frac{L_{fdfd}}{Sat_d} \qquad L_{1d1dsat} = \frac{L_{1d1d}}{Sat_d}$$

$$L_{mqsat} = \frac{L_{mq}}{Sat_q} \qquad L_{1q1qsat} = \frac{L_{1q1q}}{Sat_q} \qquad L_{2q2qsat} = \frac{L_{2q2q}}{Sat_q}$$

This directly results in all the reactance terms being similarly scaled as follows.

$$X_{mdsat} = \frac{X_{md}}{Sat_d} \qquad X_{fdsat} = \frac{X_{fd}}{Sat_d} \qquad X_{1dsat} = \frac{X_{1d}}{Sat_d}$$

$$X_{mqsat} = \frac{X_{mq}}{Sat_q} \qquad X_{1qsat} = \frac{X_{1q}}{Sat_q} \qquad X_{2qsat} = \frac{X_{2q}}{Sat_q}$$

As examples, the following carries this through calculating saturated values of the X_d'' and T_{do}'' terms. They are as follows and then expanded.

$$X_d'' = X_l + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{lfd}} + \frac{1}{X_{l1d}}} \qquad T_{do}'' = \frac{1}{\omega_s R_{1d}}\left(X_{l1d} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{lfd}}}\right)$$

$$X_d'' = X_l + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{fd} - X_{md}} + \frac{1}{X_{1d} - X_{md}}} \qquad T_{do}'' = \frac{1}{\omega_s R_{1d}}\left(X_{1d} - X_{md} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{fd} - X_{md}}}\right)$$

Now apply saturation to the appropriate reactance terms and then simplify.

$$X''_{dsat} = X_l + \frac{1}{\frac{1}{\frac{X_{md}}{Sat_d}} + \frac{1}{\frac{X_{fd}}{Sat_d} - \frac{X_{md}}{Sat_d}} + \frac{1}{\frac{X_{1d}}{Sat_d} - \frac{X_{md}}{Sat_d}}} \qquad T''_{dosat} = \frac{1}{\omega_s R_{1d}}\left(\frac{X_{1d}}{Sat_d} - \frac{X_{md}}{Sat_d} + \frac{1}{\frac{1}{\frac{X_{md}}{Sat_d}} + \frac{1}{\frac{X_{fd}}{Sat_d} - \frac{X_{md}}{Sat_d}}}\right)$$

$$X''_{dsat} = X_l + \frac{1}{Sat_d}\underbrace{\left(\frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{fd} - X_{md}} + \frac{1}{X_{1d} - X_{md}}}\right)}_{X''_d - X_l} \qquad T''_{dosat} = \frac{1}{Sat_d}\underbrace{\left[\frac{1}{\omega_s R_{1d}}\left(X_{1d} - X_{md} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{fd} - X_{md}}}\right)\right]}_{T''_{do}}$$

This can be expressed more simply without all the intermediate terms as

$$X''_{dsat} = X_l + \frac{(X''_d - X_l)}{Sat_d} \qquad T''_{dosat} = \frac{T''_{dosat}}{Sat_d}$$

You can go through the same procedure to write all the saturated reactance terms which have the same form as above, as well as the saturated time constants. Also notice in equation 3.139 of the Sauer/Pai book that the normalization of E_{fd} is proportional to X_{md} which will saturate. Similarly, the scaled field current $L_{ad}I_{fd}$ is also be subject to saturation. This gives us the following saturated values.

$$X_{dsat} = \frac{X_d - X_l}{Sat_d} + X_l \qquad X_{qsat} = \frac{X_q - X_l}{Sat_q} + X_l$$

$$X'_{dsat} = \frac{X'_d - X_l}{Sat_d} + X_l \qquad X'_{qsat} = \frac{X'_q - X_l}{Sat_q} + X_l$$

$$X''_{dsat} = \frac{X''_d - X_l}{Sat_d} + X_l \qquad X''_{qsat} = \frac{X''_q - X_l}{Sat_q} + X_l$$

$$T'_{dosat} = \frac{T'_{do}}{Sat_d} \qquad T''_{dosat} = \frac{T''_{do}}{Sat_d}$$

$$T'_{qosat} = \frac{T'_{qo}}{Sat_d} \qquad T''_{qosat} = \frac{T''_{qo}}{Sat_d}$$

$$E_{fdsat} = \frac{E_{fd}}{Sat_d} \qquad L_{ad}I_{fdsat} = Sat_d L_{ad} I_{fd}$$

Note that the equations for X''_{dsat} and X''_{qsat} are identical to those which were used in the network boundary equations for GENPTF and GENTPJ. If we make these assumptions for network boundary equations of GENROU then we get the same treatment as GENTPF and GENTPJ. We can then also apply this to the block diagram for GENROU. There are 5 reactance blocks on both the d-axis and the q-axis portion of the GENROU block diagram. The following table shows how each block is translated to make the GENQEC dynamic equations block diagram.

Block on GENROU	Convert to Saturated Version	Intermediate Derivation Steps	Translates GENQEC
$X'_d - X_l$	$X'_{dsat} - X_l$	$\left[\frac{X'_d - X_l}{Sat_d} + X_l\right] - X_l$	$\frac{X'_d - X_l}{Sat_d}$
$X_d - X'_d$	$X_{dsat} - X'_{dsat}$	$\left[\frac{X_d - X_l}{Sat_d} + X_l\right] - \left[\frac{X'_d - X_l}{Sat_d} + X_l\right] = \frac{X_d - X_l}{Sat_d} - \frac{X'_d - X_l}{Sat_d}$	$\frac{X_d - X'_d}{Sat_d}$
$\frac{X'_d - X''_d}{X'_d - X_l}$	$\frac{X'_{dsat} - X''_{dsat}}{X'_{dsat} - X_l}$	$\frac{\left[\frac{X'_d - X_l}{Sat_d} + X_l\right] - \left[\frac{X''_d - X_l}{Sat_d} + X_l\right]}{\left[\frac{X'_d - X_l}{Sat_d} + X_l\right] - X_l} = \frac{\frac{X'_d - X''_d}{Sat_d}}{\frac{X'_d - X_l}{Sat_d}}$	$\frac{X'_d - X''_d}{X'_d - X_l}$
$\frac{X''_d - X_l}{X'_d - X_l}$	$\frac{X''_{dsat} - X_l}{X'_{dsat} - X_l}$	$\frac{\left[\frac{X''_d - X_l}{Sat_d} + X_l\right] - X_l}{\left[\frac{X'_d - X_l}{Sat_d} + X_l\right] - X_l} = \frac{\frac{X''_d - X_l}{Sat_d}}{\frac{X'_d - X_l}{Sat_d}}$	$\frac{X''_d - X_l}{X'_d - X_l}$
$\frac{X'_d - X''_d}{(X'_d - X_l)^2}$	$\frac{X'_{dsat} - X''_{dsat}}{(X'_{dsat} - X_l)^2}$	$\frac{\left[\frac{X'_d - X_l}{Sat_d} + X_l\right] - \left[\frac{X''_d - X_l}{Sat_d} + X_l\right]}{\left(\left[\frac{X'_d - X_l}{Sat_d} + X_l\right] - X_l\right)^2} = \frac{\frac{X'_d - X''_d}{Sat_d}}{\left(\frac{X'_d - X_l}{Sat_d}\right)^2}$	$\frac{X'_d - X''_d}{(X'_d - X_l)^2} Sat_d$

The exact same thing can be done on the q-axis and simply adds some multiplication or division by Sat_q instead. The conversion for the time constant and field voltage term are direct as they are just scaled by the saturation function. This gives you the following block diagram.