Revisiting
Three-Phase Induction Machine

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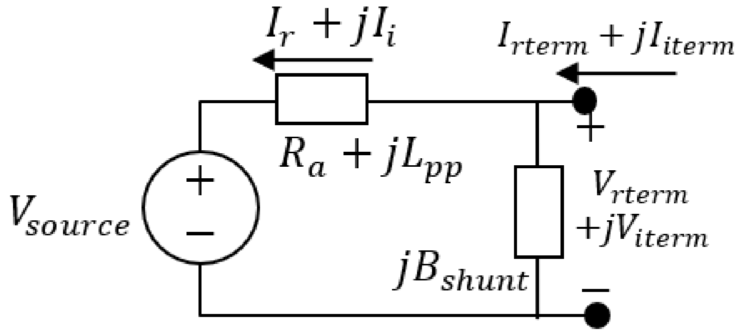
What Prompted Revisiting?

- Dmitry Kosterev of the Bonneville Power Administration (BPA) was working with Kannan Sreenivasachar from ISO-New England (ISONE)
  - Kannan was noticing that as he decreased the frequency in the system, the steady state electric power of induction motor load was not impacted by frequency changes
- Thus as a new steady-state is reached, a constant *torque* mechanical load results in a constant *power* electrical load
  - Thus electrical frequency drop does not impact electrical power at steady state.
- Dmitry emailed me to ask why this was occurring
  - My response initially was "that's what is supposed to happen"
    - I had noticed this in the equations coded for a three-phase induction motor 10 years ago, and just assumed it was fine
  - BPA, ISONE, and EPRI have experience testing hardware, and to all of them this did not seem right
  - They tested in 4 different software tools and all showed the same response
  - Time to revisit induction machine theory!

Network Interface for Models

- Differential equations give output of ***fluxes***
- The network interface equation is a voltage relationship, so we should multiply by the *stator electrical frequency* in per unit ω_{bus}
- This term was missing in traditional software implementation

$$V_{source} = \omega_{bus}(E_{ppr} + jE_{ppi})$$



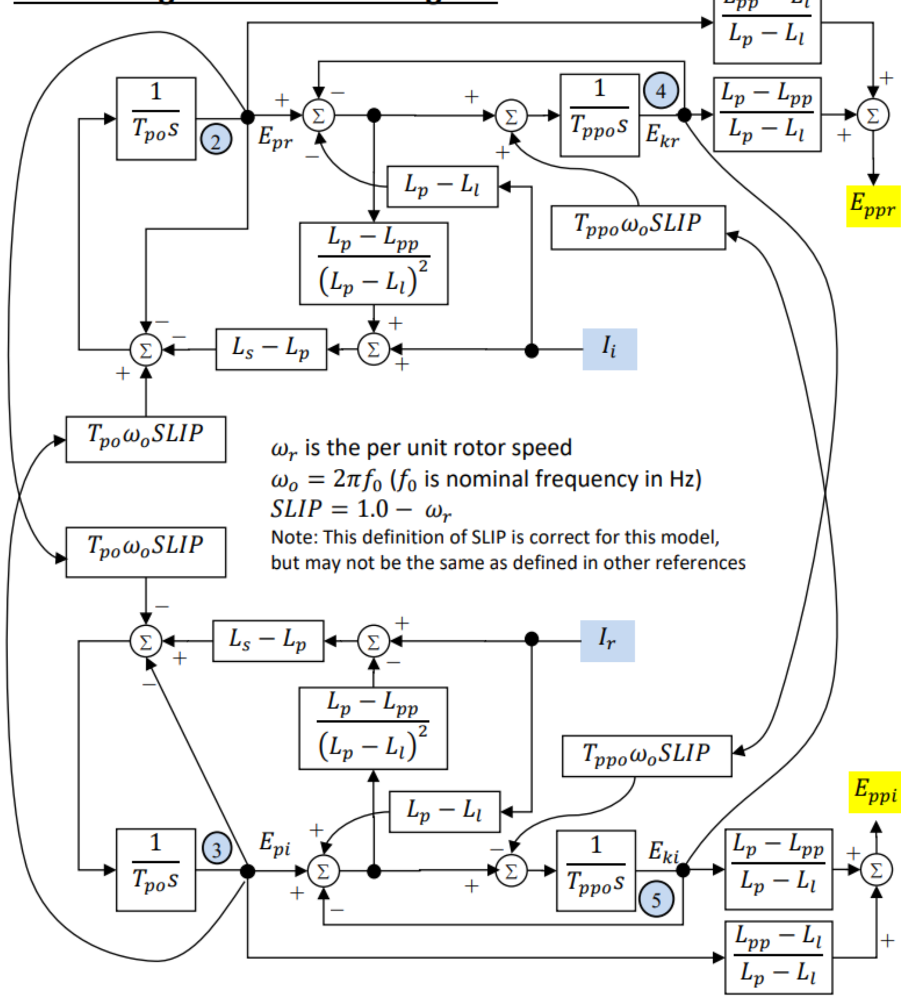
B_{shunt} is determined as part of model initialization

This fixes our problem

- With multiplication added, a constant *__torque__* load will no longer behave as a constant electric *__power__* load at the network level
- PowerWorld Simulator has added a new model named **InductionMotor3P_A**
- https://www.powerworld.com/WebHelp/#TransientModels_HTML/Load%20Characteristic%20InductionMotor3P_A.htm

InductionMotor3P_A

Double-Cage Motor Block Diagram



ω_r is the per unit rotor speed

$\omega_o = 2\pi f_0$ (f_0 is nominal frequency in Hz)

$SLIP = 1.0 - \omega_r$

Note: This definition of SLIP is correct for this model, but may not be the same as defined in other references

Mechanical Equation

$T_{elec} = E_{ppr}I_r + E_{ppi}I_i$

(1) $\frac{d\omega_r}{dt} = \frac{1}{2H}(T_{elec} - T_{nom}\omega_r^{Etrq})$

Network Interface Equations

$StatorCurrent = I_r + jI_i$

$Z_{source} = R_s + jL_{pp}$

$Y_{source} = \frac{1}{R_s + jL_{pp}} = G + jB$

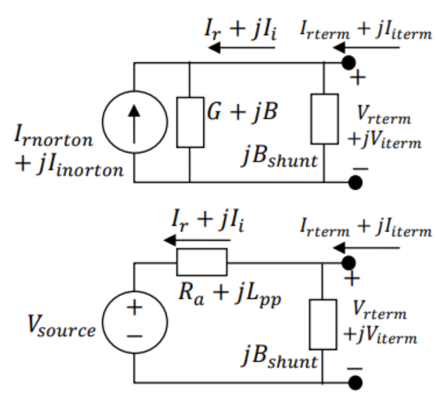
ω_{bus} = terminal bus frequency in per unit

If $flag \leq 0$

Then $V_{source} = (E_{ppr} + jE_{ppi})$

Else $V_{source} = \omega_{bus}(E_{ppr} + jE_{ppi})$

$I_{rnorton} + jI_{inorton} = V_{source}(G + jB)$



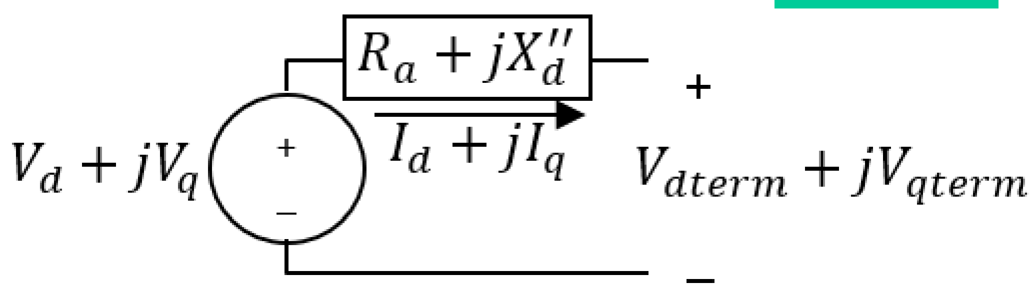
B_{shunt} is determined as part of model initialization

Flag parameter indicates whether to include this multiplication

Similar to Synchronous Machine

- Same concept is in the synchronous machine models too
- GENROU and related models have multiplied by **rotor speed** instead
    - ω = rotor speed deviation in per unit

$$V_d + jV_q = \left(-\psi_q'' + j\psi_d''\right)(1+\omega)$$



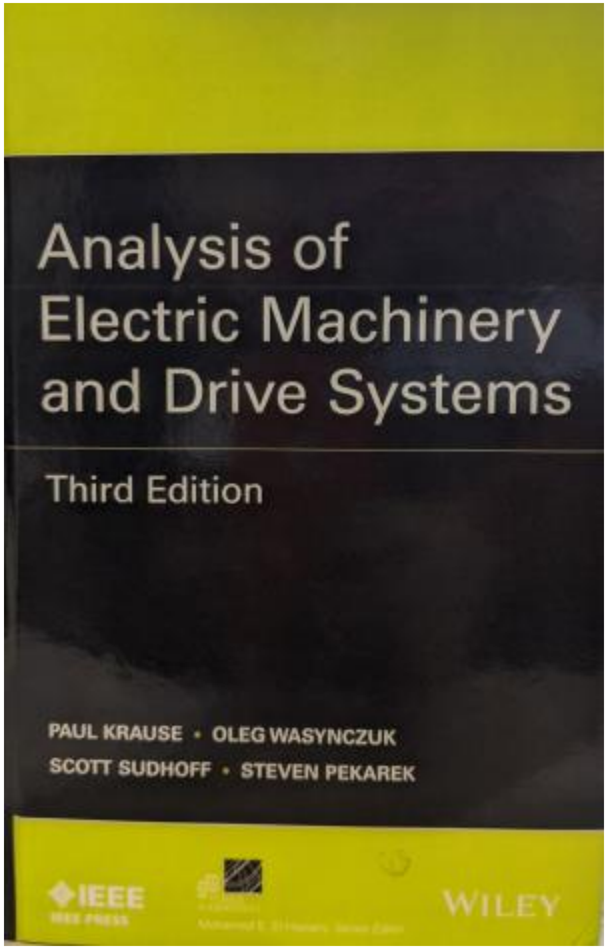
- Should we be using the stator electrical frequency for a synchronous machine too?
    – We don't need to because $\omega_r \rightarrow \omega_e$ in synchronous machine this approximation is fine for synchronous machine

Comparing Induction Machine and Synchronous Machine

	Synchronous Machine	**Induction Machine**
Transient Time Frame	Small deviations in rotor speed during simulation	Large deviations in rotor speed during simulation
Steady State	Rotor speed and stator electric speed are equal at steady state	Rotor speed is not equal to electric speed at steady state (always a slip)
Is using Rotor Speed instead of stator electric speed a good approximation?	YES, Good approximation	NO, Must use stator frequency in per unit

Proving this with Math: Krause Book

- Another book *Analysis of Electric Machine and Drive Systems*, Paul Krause, Oleg Wasynczuk, Scott Sudhoff, Steven Pekarek
  - First Edition published in 2003
  - Third Edition is 2013



Proof that Stator Electric Frequency should be used

- Start with Krause et. All equation 6.5-22 and 6.5-23 on page 227 of the book
- These equations are on an ***arbitrary*** reference frame (ω has not been chosen)

$$v_{qs} = r_s i_{qs} + \frac{\omega}{\omega_b}\psi_{ds} + \frac{1}{\omega_b}\frac{d\psi_{qs}}{dt} \qquad 6.5\text{-}22$$

$$v_{ds} = r_s i_{ds} - \frac{\omega}{\omega_b}\psi_{qs} + \frac{1}{\omega_b}\frac{d\psi_{ds}}{dt} \qquad 6.5\text{-}23$$

- On the rotor reference frame gives

$$v_{qs} = r_s i_{qs} + \frac{\omega_r}{\omega_b}\psi_{ds} + \frac{1}{\omega_b}\frac{d\psi_{qs}}{dt}$$

$$v_{ds} = r_s i_{ds} - \frac{\omega_r}{\omega_b}\psi_{qs} + \frac{1}{\omega_b}\frac{d\psi_{ds}}{dt}$$

Traditional Simplification ignoring stator transients

- Simplification for ignoring stator transients traditionally done is to ignore the derivatives of stator flux

$$v_{qs} = r_s i_{qs} + \frac{\omega_r}{\omega_b}\psi_{ds} + \frac{1}{\omega_b}\frac{d\psi_{qs}}{dt}$$

$$v_{ds} = r_s i_{ds} - \frac{\omega_r}{\omega_b}\psi_{qs} + \frac{1}{\omega_b}\frac{d\psi_{ds}}{dt}$$

- In synchronous machine we often assume those terms go quickly to zero and jump directly to

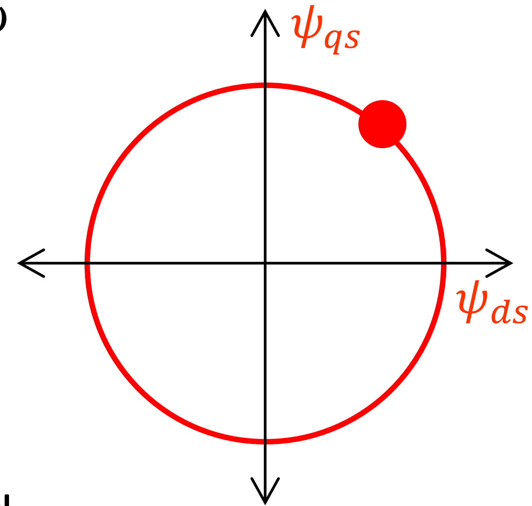
$$v_{qs} = r_s i_{qs} + \frac{\omega_r}{\omega_b}\psi_{ds}$$

$$v_{ds} = r_s i_{ds} - \frac{\omega_r}{\omega_b}\psi_{qs}$$

- This is a valid approximation for a *synchronous* machine in the *rotor* reference frame
  - This feels like it's always correct, but it depends on the reference frame and the type of machine

Do $d\psi_{qs}/dt$ and $d\psi_{qs}/dt$ always quickly go to zero? NO!

- Depends on the reference frame and the machine type
  - Magnitude reaches new steady state quickly with $\frac{d\psi_s}{dt} \rightarrow 0$
  - ψ_{qs} and ψ_{ds} reach a steady state spinning around a circle with a frequency of $\omega_e - \omega$
  - $\frac{d\psi_{qs}}{dt}$ and $\frac{d\psi_{ds}}{dt}$ derivatives become sinusoids and do NOT go to zero
- In rotor reference frame $\omega = \omega_r$, so that new steady state has the fluxes states spinning at $\omega_e - \omega_r$
  - In a synchronous machine, the new steady state will have $\omega_e = \omega_r$, thus that means the red dot becomes fixed
  - For synchronous machines it is correct that $\frac{d\psi_{qs}}{dt}$ and $\frac{d\psi_{ds}}{dt}$ would reach a new steady state and would go toward 0 quickly
  - This is NOT true for all machines and references



Need to go back to the ABC phase phasors

ABC Phase Quantities

$$f_{as} = \sqrt{2} f_s\, cos(\theta_e)$$

$$f_{bs} = \sqrt{2} f_s\, cos\left(\theta_e - \frac{2\pi}{3}\right)$$

$$f_{cs} = \sqrt{2} f_s\, cos\left(\theta_e + \frac{2\pi}{3}\right)$$

$$\omega_e = \frac{d\theta_e}{dt}$$

After DQ transformation in an Arbitrary Reference Frame

$$f_{qs} = +\sqrt{2} f_s \cos(\theta_e - \theta)$$

$$f_{ds} = -\sqrt{2} f_s \sin(\theta_e - \theta)$$

$$f_{os} = 0$$

$$\omega = \frac{d\theta}{dt}$$

$$f_s = \frac{\sqrt{f_{qs}^2 + f_{ds}^2}}{\sqrt{2}}$$

Now Take Derivatives

$$f_{qs} = +\sqrt{2} f_s \cos(\theta_e - \theta)$$

$$f_{ds} = -\sqrt{2} f_s \sin(\theta_e - \theta)$$

$$\frac{df_{qs}}{dt} = \frac{d}{dt}\left[\sqrt{2} f_s\, cos(\theta_e - \theta)\right]$$
$$\frac{df_{qs}}{dt} = \frac{df_s}{dt}\left[\sqrt{2}\, cos(\theta_e - \theta)\right] - \sqrt{2} f_s\, sin(\theta_e - \theta)\left(\frac{d\theta_e}{dt} - \frac{d\theta}{dt}\right)$$
$$\frac{df_{qs}}{dt} = \frac{df_s}{dt}\left[\frac{\sqrt{2} f_s\, cos(\theta_e - \theta)}{f_s}\right] + \left[-\sqrt{2} f_s\, sin(\theta_e - \theta)\right]\left(\frac{d\theta_e}{dt} - \frac{d\theta}{dt}\right)$$
$$\frac{df_{qs}}{dt} = \frac{df_s}{dt}\frac{f_{qs}}{f_s} + f_{ds}(\omega_e - \omega)$$

$$\frac{df_{ds}}{dt} = \frac{d}{dt}\left[-\sqrt{2} f_s\, sin(\theta_e - \theta)\right]$$
$$\frac{df_{ds}}{dt} = \frac{df_s}{dt}\left[-\sqrt{2} f_s\, sin(\theta_e - \theta)\right] - \sqrt{2} f_s\, cos(\theta_e - \theta)\left(\frac{d\theta_e}{dt} - \frac{d\theta}{dt}\right)$$
$$\frac{df_{ds}}{dt} = \frac{df_s}{dt}\left[\frac{-\sqrt{2} f_s\, sin(\theta_e - \theta)}{f_s}\right] + \left[-\sqrt{2} f_s\, cos(\theta_e - \theta)\right]\left(\frac{d\theta_e}{dt} - \frac{d\theta}{dt}\right)$$
$$\frac{df_{ds}}{dt} = \frac{df_s}{dt}\frac{f_{ds}}{f_s} - f_{qs}(\omega_e - \omega)$$

Apply this to our equations in the *Arbitrary* Reference Frame

$$v_{qs} = r_s i_{qs} + \frac{\omega}{\omega_b}\psi_{ds} + \frac{1}{\omega_b}\frac{d\psi_{qs}}{dt}$$
$$v_{ds} = r_s i_{ds} - \frac{\omega}{\omega_b}\psi_{qs} + \frac{1}{\omega_b}\frac{d\psi_{ds}}{dt}$$

$$\frac{d\psi_{qs}}{dt} = \frac{\psi_{qs}}{\psi_s}\frac{d\psi_s}{dt} + \psi_{ds}(\omega_e - \omega)$$
$$\frac{d\psi_{ds}}{dt} = \frac{\psi_{ds}}{\psi_s}\frac{d\psi_s}{dt} - \psi_{qs}(\omega_e - \omega)$$

Substitute in our derivative calculation

$$v_{qs} = r_s i_{qs} + \frac{\omega}{\omega_b}\psi_{ds} + \frac{1}{\omega_b}\left[\frac{\psi_{qs}}{\psi_s}\frac{d\psi_s}{dt} + \psi_{ds}(\omega_e - \omega)\right]$$
$$v_{ds} = r_s i_{ds} - \frac{\omega}{\omega_b}\psi_{qs} + \frac{1}{\omega_b}\left[\frac{\psi_{ds}}{\psi_s}\frac{d\psi_s}{dt} - \psi_{qs}(\omega_e - \omega)\right]$$

Expand Terms

$$v_{qs} = r_s i_{qs} + \frac{\omega}{\omega_b}\psi_{ds} + \frac{\omega_e}{\omega_b}\psi_{ds} - \frac{\omega}{\omega_b}\psi_{ds} + \frac{1}{\omega_b}\left[\frac{\psi_{qs}}{\psi_s}\frac{d\psi_s}{dt}\right]$$
$$v_{ds} = r_s i_{ds} - \frac{\omega}{\omega_b}\psi_{qs} - \frac{\omega_e}{\omega_b}\psi_{qs} + \frac{\omega}{\omega_b}\psi_{qs} + \frac{1}{\omega_b}\left[\frac{\psi_{ds}}{\psi_s}\frac{d\psi_s}{dt}\right]$$

Cancel Terms

$$v_{qs} = r_s i_{qs} + \frac{\omega_e}{\omega_b}\psi_{ds} + \frac{1}{\omega_b}\left[\frac{\psi_{qs}}{\psi_s}\frac{d\psi_s}{dt}\right]$$
$$v_{ds} = r_s i_{ds} - \frac{\omega_e}{\omega_b}\psi_{qs} + \frac{1}{\omega_b}\left[\frac{\psi_{ds}}{\psi_s}\frac{d\psi_s}{dt}\right]$$

Ignore Stator Transients in any Reference Frame or Machine

- Approximation to ignore stator transients is

$$v_{qs} = r_s i_{qs} + \frac{\omega_e}{\omega_b}\psi_{ds} + \frac{1}{\omega_b}\left[\frac{\psi_{qs}}{\psi_s}\frac{d\psi_s}{dt}\right]$$
$$v_{ds} = r_s i_{ds} - \frac{\omega_e}{\omega_b}\psi_{qs} + \frac{1}{\omega_b}\left[\frac{\psi_{ds}}{\psi_s}\frac{d\psi_s}{dt}\right]$$

These terms quickly go to zero regardless of reference frame choice and machine type

- Result for an *arbitrary* reference frame

$$v_{qs} = r_s i_{qs} + \frac{\omega_e}{\omega_b}\psi_{ds}$$
$$v_{ds} = r_s i_{ds} - \frac{\omega_e}{\omega_b}\psi_{qs}$$

  - Use this for our induction machine!
  - Could also use this for a synchronous machine
    - Also valid to use $\frac{\omega_r}{\omega_b}$ for a synchronous machine though, so we do that because it's easier (ω_r is a state)